Three Dollars per Annum, in Advance. Single Copies, Twenty-Five Cents.

Vol. II. No. II.

THE

MATHEMATICAL MONTHLY.

NOVEMBER, 1859.

EDITED BY

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NEW YORK:
PUBLISHED BY IVISON & PHINNEY.
LONDON:
TRÜBNER AND CO.
1859.

MATHEMATICAL MONTHLY PRIZES.

We are informed that some of the conditions under which these Prizes are offered are not well understood; especially by those whose attention was first called to the matter in the last number of the Monthly. There are two classes of Prizes; I. For Solutions. In each number of the Monthly we give five problems, entitled *Prize Problems for Students*. The first and second of these problems are quite simple, and are intended for students in High Schools, Academies, and all Institutions not conferring degrees; and for the best solutions of which a *copy of the Monthly is given* as a prize. The third, fourth, and fifth problems are somewhat more difficult, and are open to the competition of all students, whether connected with an Educational Institution or not. For the solution of these problems, two prizes are offered; a first prize of *six dollars*, and a second prize of *four dollars*. Conditions: All the steps in each solution must be fully given; cuts, if any, must be neatly drawn, of the proper size, and the whole communicated in a plain and legible hand-writing. The name of the competitor, as well as the Institution with which he is connected, if any, must be written on a separate slip, which must also be signed by his Teacher, as evidence that the party is fairly entitled to compete for the prize. Credit will in all cases be given for all solutions received. II. For Essays: Five prizes are given for essays; a first prize of *fifty dollars;* a second prize of *forty dollars;* a third prize of *thirty dollars;* a fourth prize of *twenty dollars;* and a fifth prize of *ten dollars*. The first and second of these prizes are open to all competitors; the third, fourth, and fifth are open to all students, and under the same conditions as the prizes for the third, fourth, and fifth problems. . . . . Our contributors will much oblige us by using, as far as possible, the cuts already engraved for the Monthly. In many cases a few more lines and letters will adapt the cut to the new problem; and these additions can be readily made, thus saving us the trouble and expense of engraving an entirely new cut.

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Entered according to Act of Congress in the year 1859, by

J. D. RUNKLE,

In the Clerk's Office of the District Court of the District of Massachusetts.

TERMS.

(INVARIABLY IN ADVANCE.)

A single Copy,	$3.00 per annum.
Two Copies to one Address,	5.00 "
Five Copies " "	11.00 "
Ten Copies " "	20.00 "

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IVISON & PHINNEY, *Publishers, New York.*

W. P. Brown, Book and Job Printer, 421 Main Street, Cambridgeport.

PRESS OF WELCH, BIGELOW & COMPANY, CAMBRIDGE.

Nath. Bowditch

THE

MATHEMATICAL MONTHLY.

Vol. II.. NOVEMBER, 1859. No. II.

PRIZE PROBLEMS FOR STUDENTS.

I. Form the equation, the roots of which are $a+\sqrt{b}, a-\sqrt{b}, -c$ and d

II. Find the equated time of payment of two sums S and s, due respectively at the end of T and t years, allowing simple interest.

III. Of all right-angled plane triangles having the same given hypothenuse, to find the one whose area is the greatest possible. To be solved by Algebra.

IV. What is that fraction, the cube of which being subtracted from it, the remainder is the greatest possible? To be solved by Algebra.

V. If, in a plane or spherical triangle, A, B, C denote the angles, and a, b, c the sides respectively opposite them; and if we produce the sides of the triangle, and consider the three circles which touch two of the sides interiorly and the third side exteriorly; and denote by r, ϱ the radii of the circumscribed and inscribed circles; by $\varrho', \varrho'', \varrho'''$, the radii of the circles touching exteriorly the sides a, b, c respectively; by $\delta', \delta'', \delta'''$, the distances of the centres of

these circles from the centre of the circle circumscribed about the primitive triangle; then we have in the plane

$$-\sin A+\sin B+\sin C=4\cos\tfrac{1}{2}A\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C,$$
$$\sin A-\sin B+\sin C=4\sin\tfrac{1}{2}A\cos\tfrac{1}{2}B\sin\tfrac{1}{2}C,$$
$$\sin A+\sin B-\sin C=4\sin\tfrac{1}{2}A\sin\tfrac{1}{2}B\cos\tfrac{1}{2}C,$$

and in the sphere

$$-\sin A+\sin B+\sin C=4\cos\tfrac{1}{2}A\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C.\frac{\cos\delta'}{\cos r\cos\varrho'},$$
$$\sin A-\sin B+\sin C=4\sin\tfrac{1}{2}A\cos\tfrac{1}{2}B\sin\tfrac{1}{2}C.\frac{\cos\delta''}{\cos r\cos\varrho''},$$
$$\sin A+\sin B-\sin C=4\sin\tfrac{1}{2}A\sin\tfrac{1}{2}B\cos\tfrac{1}{2}C.\frac{\cos\delta'''}{\cos r\cos\varrho'''}.$$

The solution of these problems must be received by the first of January, 1860. Problem V. is one of the analogies by Prof. Chauvenet, to which we referred in the last number of the Monthly.

Report of the Judges upon the Solutions of the Prize Problems in No. X., Vol. I.

The first Prize is awarded to Gustavus Frankenstein, of Springfield, Ohio.

The second Prize is awarded to Roland Thompson, Jefferson College, Canonsburg, Pa.

Prize Solution of Problem II.

BY ALL THE COMPETITORS.

"If A, B, C be the angles, and a, b, c the opposite sides, in a plane triangle, of which S denotes the surface; prove that

$$a^2+b^2+c^2=4S(\cot A+\cot B+\cot C).$$"

By Trigonometry,

$$a^2=b^2+c^2-2bc\cos A$$

$$b^2 = a^2 + c^2 - 2\,a\,c\cos B$$
$$c^2 = a^2 + b^2 - 2\,a\,b\cos C.$$

By addition and transposition we get

(1) $\quad a^2 + b^2 + c^2 = 2\,(b\,c\cos A + a\,c\cos B + a\,b\cos C).$

But $2\,S = b\,c\sin A = a\,c\sin B = a\,b\sin C$:

or $$b\,c = \frac{2\,S}{\sin A}, \quad a\,c = \frac{2\,S}{\sin B}, \quad a\,b = \frac{2\,S}{\sin C}.$$

Substituting these values of $b\,c$, $a\,c$, $a\,b$, in (1), we obtain

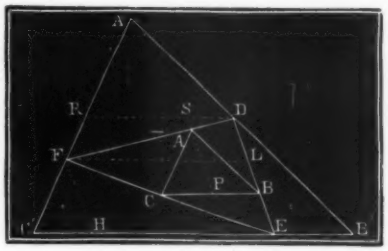
$$a^2 + b^2 + c^2 = 4\,S(\cot A + \cot B\,\cot C).$$

Prize Solution of Problem III.

BY GUSTAVUS FRANKENSTEIN, SPRINGFIELD, OHIO.

"If one of the similar triangles $A\,B\,C$ and $A'\,B'\,C'$ be inscribed in the triangle $D\,E\,F$ and the other circumscribed about it; prove that the area of $D\,E\,F$ will be a mean proportional between the areas of $A\,B\,C$ and $A'\,B'\,C'$."

Let $D\,E\,F$ be any triangle, $A\,B\,C$ an inscribed, and $A'B'C'$ a similar circumscribed, triangle. Draw $D\,S\,R$ and $F\,L$ parallel to $B\,C$ or $B'C'$; also, $F\,H$ and $S\,A\,P$ parallel to $D\,E$. By reason of these parallels, the triangle $B'E\,D$ is similar to $B\,P\,A$, $E\,B\,C$ to $E\,L\,F$, $F\,L\,D$ to $D\,S\,A$, and $C'\,H\,F$ to $C\,P\,A$; whence the proportions:



(1) . . . $B'E : E\,D = B\,P : P\,A$ (2) . . . . $E\,B : B\,C = E\,L : L\,F$

(3) . . . . $L\,F : D\,L = S\,D : S\,A$ (4) . . . . $C'H : H\,F = P\,C : P\,A$;

in which

$$F\,H = L\,E = D\,E - D\,L, \quad S\,D = P\,B, \quad B\,C = B\,P + P\,C,$$
$$H\,E = L\,F, \quad S\,A + A\,P + B\,E = D\,E.$$

But (2) and (4) give

$$(D\,E - D\,L)(B\,P + P\,C) = E\,B \times L\,F, (D\,E - D\,L)\,P\,C = C'H \times P\,A;$$

hence $(BP+PC)\times C'H\times PA=PC\times EB\times LF$,

or, $PC\times(EB\times FL-C'H\times PA)=BP\times C'H\times PA$;

and as $PC\times(DE-DL)=C'H\times PA$,

therefore $(DE-DL)BP=(EB\times LF-C'H\times PA)$.

Substitute for DE its value $\frac{B'E\times PA}{BP}$ deduced from (1), and for DL its value $\frac{FL\times SA}{BP}$ deduced from (3), and there results the condition,

(5) $B'E\times PA-FL\times SA=EB\times LF-C'H\times PA$,

or $(B'E+C'H)PA=(EB+SA)FL$.

Add $EH\times PA$ to the first side of this equation, and its equal $FL\times PA$ to the second; then, because

$$B'E+EH+C'H=B'C',\ SA+AP+BE=DE,$$

(5) becomes

(6) $B'C'\times PA=DE\times LF$.

But in the triangles ABC and DEF, the angle $APB=FLE$;

(7) $\therefore$ $ABC:DEF=AP\times BC:FL\times DE$;

or, denoting the area of the triangle ABC by t, and by (6),

$$t:DEF=AP\times BC:B'C'\times AP,$$

$$\therefore DEF=t\times\frac{B'C'\times AP}{BC\times AP}=t\times\frac{B'C'}{BC};$$

Also, $ABC:A'B'C'=BC^2:B'C'^2$;

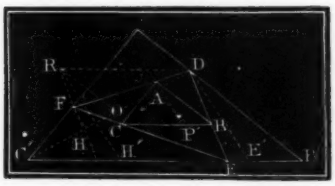
or $A'B'C'=t\times\frac{B'C'^2}{BC^2}$

$$\therefore ABC\times A'B'C'=t^2\times\frac{B'C'^2}{BC^2}=(DEF)^2.$$

Therefore DEF is a mean proportional between ABC and $A'B'C'$.

Corollary. The property $B'C'\times PA=DE\times(LF$ or $EH)$ is not confined to the side DE of the triangle DEF, and its par-

allel AP, but applies to any line DE' and its parallel AP'. For



the triangle DEF is half of the parallelogram DH; having the same base and altitude; and for the same reason the parallelogram $DH = DH'$. Hence, as the angle $AP'C = H'E'D$,

$$\therefore \quad t : DEF = BC \times AP' : (HE \text{ or } H'E')\, DE',$$

$$\therefore \quad DEF = t \times \frac{HE \times DE'}{BC \times AP'},$$

$$\therefore \quad (DEF)^2 = t^2 \times \frac{HE^2 \times DE'^2}{BC^2 \times AP'^2} = t^2 \times \frac{B'C'^2}{BC^2},$$

$$\therefore \quad HE \times DE' = B'C' \times AP'.$$

Therefore the line DE' may take the direction DB', and we shall have

$$B'C' \times AB = DB' \times EH;$$

so that drawing $C'D$, and RO parallel to DB', intersecting $C'D$ in O, we must have $RO = AB$. For the triangles $C'DB'$ and DRO are similar, having their sides respectively parallel, giving

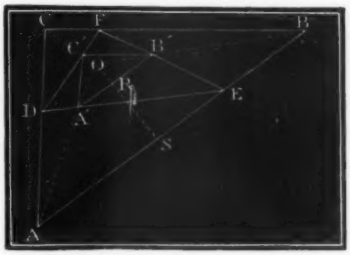
$$B'C' : B'D = RD \text{ or } EH : RO.$$

$$\therefore\ DB' \times EH = B'C' \times RO = B'C' \times AB. \quad \therefore\ RO = AB.$$

Second Solution of Prize Problem III.

by roland thompson, jefferson college, canonsburg, pa.

Inscribe the triangle $A'B'C'$ in DEF, and through the points D, E, F, draw straight lines respectively parallel to the sides of $A'B'C'$; the triangle ABC thus formed is similar to $A'B'C'$. The three straight lines passing through the homologous angles will meet in the same point. For the straight lines CC' and BB' produced will meet in some point



O, and if AO does not pass through A', let A'' be the point in which it meets the line $A'B'$. Then

$$BO : B'O = AB : A''B'$$
$$BO : B'O = BC : B'C';$$
$$\therefore \quad AB : A''B' = BC : B'C'.$$

Therefore the triangle $A''B'C'$ is similar to ABC and equal to $A'B'C'$, and therefore $A''B' = A'B'$, and AO passes through A'. Let OS be drawn perpendicular to AB, and $A'B'$; then, since OS, OR, and RS, are the altitudes of the triangles AOB, $A'OB'$, and $A'B'E$, we have

$$(1) \quad \frac{AOB}{A'OB'E} = \frac{AOB}{A'OB' + A'B'E} = \frac{AB \times OS}{A'B' \times OR + A'B' \times RS}$$
$$= \frac{AB \times OS}{A'B' \times OS} = \frac{AB}{A'B'} = \frac{OS}{OR} = \frac{A'OB'E}{A'OB'}.$$

In like manner

$$(2) \quad \frac{BOC}{B'FC'O} = \frac{B'FC'O}{B'OC'} = \frac{BC}{B'C'} = \frac{AB}{A'B'}.$$

$$(3) \quad \frac{AOC}{A'OC'D} = \frac{A'OC'D}{A'OC'} = \frac{AC}{A'C'} = \frac{AB}{A'B'}.$$

Since (1), (2), and (3), are equal ratios, we get, by addition,

$$\frac{ABC}{DEF} = \frac{DEF}{A'B'C'},$$

showing that DEF is a mean proportional between ABC and $A'B'C'$ as was to be proved.

Prize Solution of Problem IV.

BY ALL THE COMPETITORS.

"If a be one of the sides of an equilateral spherical triangle and A one of its angles, prove that $\sec A = \sec a + 1$."

If $a = b = c$ in the fundamental equation

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

it becomes

$$\cos a = \cos^2 a + \sin^2 a \cos A.$$

$$\therefore \quad \cos A = \frac{\cos a - \cos^2 a}{\sin^2 a} = \frac{\cos a\,(1 - \cos a)}{(1 + \cos a)(1 - \cos a)} = \frac{\cos a}{1 + \cos a}.$$

$$\therefore \quad \sec A = \frac{1 + \cos a}{\cos a} = \sec a + 1.$$

Prize Solution of Problem V.

BY GUSTAVUS FRANKENSTEIN, SPRINGFIELD, OHIO.

"If the semiaxes of an ellipse be A and B, P the length of the perpendicular dropped from the centre on the tangent to the curve, r and r' the distances from the point of tangency to the foci, and ϱ the radius of curvature at this point; prove that

$$\varrho = \frac{A^2 B^2}{P^3} = \frac{r r'}{P};$$

and from this theorem construct the corresponding point of the evolute."

1. The equation of the ellipse referred to its centre and axes is $A^2x^2 + B^2y^2 = A^2B^2$; and if x', y' denote the coördinates of the point of tangency, the equation of the tangent line is $A^2 y y' + B^2 x x' = A^2 B^2$. The tangent of the angle which the line DS makes with the axis of x is $D_x y = -\frac{B^2 x'}{A^2 y'}$; or if we take the angle TSC, then $\tan TSC = \tan MTO = \frac{B^2 x'}{A^2 y'} = \alpha$.

If in the equation of the tangent line $y = 0$, then $CS = x = \frac{A^2}{x'}$; the normal $MT = N = y'\sqrt{1 + \alpha^2}$; subnormal $MO = \alpha y'$; the subtangent $SO = \frac{y'}{\alpha}$, the similar triangles TSM and DSC give $DC : CS = TM : MO + OS$;

or $$P : \frac{A^2}{x'} = N : \alpha y' + \frac{y'}{\alpha}.$$

$$\therefore \qquad P = \frac{A^2 N}{x'} \times \frac{a}{y'(1+a^2)} = \frac{A^2 N}{x'} \times \frac{a\,y'}{N^2} = \frac{B^2}{N}.$$

But from the Calculus we have the radius of curvature, $\varrho = \frac{A^2 N^3}{B^4}$;

$$\therefore \qquad \varrho \times P^3 = \frac{A^2 N^3}{B^4} \times \frac{B^6}{N^3} = A^2 B^2. \quad \therefore \varrho = \frac{A^2 B^2}{P^3}.$$

Further, the normal, MT, bisects the angle ATF;

$$\therefore \qquad \frac{r'}{r} = \frac{AM}{MF}, \frac{r'+r}{r} = \frac{2A}{r} = \frac{AM+MF}{MF} = \frac{2\sqrt{A^2-B^2}}{MF}.$$

$$\therefore \qquad MF = \frac{r\sqrt{A^2-B^2}}{A}, \text{ and } AM = \frac{r'\sqrt{A^2-B^2}}{A}.$$

Also, in the triangle ATF we have

$$AT \times TF = AM \times MF + MT^2;$$

or, $$r'r = \frac{r'r(A^2-B^2)}{A^2} + N^2. \quad \therefore r'r = \frac{A^2 N^2}{B^2}.$$

Dividing by $P = \frac{B^2}{N}$ gives $\frac{r'r}{P} = \frac{A^2 N^3}{B^4} = \varrho$.

2. To find by construction the corresponding point in the evolute. We have just found that $r' : P = \varrho : r$. Produce the normal TM, and also TF. On TM produced take $Tp = P$, and $TL = r'$. Draw pF, and LX parallel to pF; then

$$TL : Tp = TX : TF;$$

or $$r' : P = TX : r'. \quad \therefore TX = TE = \varrho.$$

This construction is by Frederic T. Hampton.

Chauncey Wright.
Truman Henry Safford.

SOLUTION OF PROBLEMS IN MAXIMA AND MINIMA BY ALGEBRA.

By Ramchundra, Late Teacher of Science, Delhi College.

1. *To divide a given number into two such parts that their product may be the greatest possible.*

Put the given number $= a$, one of the parts required $= x$, and consequently $a - x =$ the other part. Therefore $x(a - x) = ax - x^2 =$ product $=$ maximum $= r$. Therefore $x^2 - ax = -r$. Solving this quadratic equation we find

$$x = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - r}.$$

Now it is evident that r cannot be greater than $\frac{1}{4}a^2$; for if it be so, the value of x becomes impossible; therefore the product $ax - x^2$, or r, is greatest when $\frac{1}{4}a^2 = r$; therefore $x = \frac{a}{2}$.

Second Solution without Impossible Roots.

In the expression $ax - x^2$, which is to become a maximum, let $x = y + \frac{a}{2}$, where the value of y, determined by the condition of $ax - x^2$ being a maximum, will show whether it is positive, zero, or negative. We now find

$$ax - x^2 = ay + \frac{a^2}{2} - y^2 - ay - \frac{a^2}{4} = \frac{a^2}{4} - y^2,$$

which is evidently a maximum when $y = 0$. Therefore $x = \frac{a}{2}$ as before.

II. *Of all right-angled plane triangles having the same hypothenuse, to find that whose area is the greatest possible.*

Let $a =$ hypothenuse, $x =$ base, $y =$ perpendicular. Then, $x^2 + y^2 = a^2$, we shall have $y = \sqrt{a^2 - x^2}$, and consequently $\frac{xy}{2} = \frac{x}{2}\sqrt{a^2 - x^2} =$ the area of the triangle $=$ maximum, and consequently the square of the area, or $\frac{1}{4}(a^2x^2 - x^4) =$ max., and also

four times this, or $a^2x^2 - x^4 = \text{max.} = r$. Therefore $x^4 - a^2x^2 = -r$. Solving this quadratic equation, we find

$$x^2 = \frac{a^2}{2} \pm \sqrt{\frac{a^4}{4} - r},$$

and it is manifest that $a^2x^2 - x^4$, or r cannot be greater than $\frac{1}{4}a^4$; therefore when $r = \text{maximum}$, we must have $r = \frac{1}{4}a^4$, therefore $x^2 = \frac{1}{2}a^2$, and $x = \frac{a}{\sqrt{2}}$, and $y = \sqrt{a^2 - x^2} = \frac{a}{\sqrt{2}}$. Hence it appears that the right-angled plane triangle contains the greatest area whose two sides, containing the right angle, are equal to each other.

SECOND SOLUTION WITHOUT IMPOSSIBLE ROOTS.

In the expression $a^2x^2 - x^4$, which is to become a maximum, let $x^2 = y^2 + \frac{a^2}{2}$.

$$\therefore \quad a^2x^2 - x^4 = a^2\left(y^2 + \frac{a^2}{2}\right) - \left(y^2 + \frac{a^2}{2}\right)^2$$

$$= a^2y^2 + \frac{a^4}{2} - y^4 - a^2y^2 - \frac{a^4}{4} = \frac{a^4}{4} - y^4.$$

which is evidently a maximum, when $y^4 = 0$, and therefore $x^2 = \frac{a^2}{2}$, or $x = \frac{a}{\sqrt{2}}$, as before.

III. *To bisect a triangle by the shortest line.*

Let ABC be the given triangle, and DQ the shortest line required. Also, let $CD = x$, $CQ = y$, $DQ = u$, and a, b, c the three sides of the triangle, and C the angle BCA. DM and BN are perpendiculars, drawn from the points D and B, on the line CA. Now, by similar triangles, we find $\frac{DM}{CD} = \frac{BN}{CB} = \sin C$; therefore $DM = x \sin C$, and $BN = a \sin C$, and

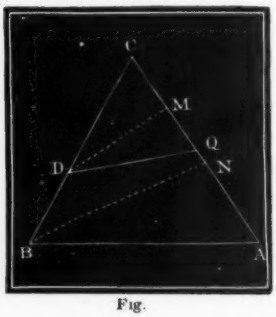


Fig.

$$\therefore \quad \frac{CQ \times DM}{2} = \frac{xy \sin C}{2}; \quad \frac{CA \times BN}{2} = \frac{ab \sin C}{2};$$

but, by supposition,

$$2 \times \frac{xy \sin C}{2} = \frac{ab \sin C}{2}; \ \therefore \ ab = 2xy; \ \therefore \ y = \frac{ab}{2x}.$$

By Proposition 13, Book 2d, of Euclid, we find

$$u^2 = x^2 + y^2 - 2xy \cos C = x^2 + \frac{a^2 b^2}{4x^2} - ab \cos C = \text{min.} = r.$$

$$\therefore \quad x^4 + \frac{a^2 b^2}{4} - ab x^2 \cos C = r x^2;$$

$$\therefore \quad x^4 - (ab \cos C + r) x^2 = -\frac{a^2 b^2}{4}.$$

Completing the square, and extracting the square root, we find,

$$x^2 = \frac{ab \cos c + r}{2} \pm \sqrt{\frac{(ab \cos C + r)^2 - a^2 b^2}{4}}.$$

Now $a^2 b^2$ is greater than $a^2 b^2 \cos^2 C$; therefore in order that the value of x^2 may not become impossible, we must have $ab \cos C + r = ab$; therefore $r = ab - ab \cos C$, and therefore, when $r = \text{min.}$, we must have

$$x^2 = \frac{ab \cos C + r}{2} = \frac{ab}{2}; \ \therefore x = \sqrt{\frac{ab}{2}}, y = \frac{ab}{2x} = \sqrt{\frac{ab}{2}}.$$

$$\therefore \quad u^2 = \frac{ab}{2} + \frac{ab}{2} - ab \cos C = ab(1 - \cos C)$$

$$= ab\left(\frac{2ab + c^2 - (a^2 + b^2)}{2ab}\right) = \frac{c^2 - (a-b)^2}{2};$$

and therefore, $u = \sqrt{\frac{(c - a + b)(c + a - b)}{2}}$.

Second Solution without Impossible Roots.

Let $ab \cos C + r = A$, and $\frac{a^2 b^2}{4} = B$; therefore the equation $x^4 - (ab \cos C + r) x^2 = -\frac{a^2 b^2}{4}$ becomes $x^4 - A x^2 = -B$.

Also, let $x^2 = y + \frac{A}{2}$,

$$\therefore \quad x^4 - A x^2 = y^2 + Ay + \frac{A^2}{4} - Ay - \frac{A^2}{2}$$

$$= y^2 - \frac{A^2}{4} = -B. \ \therefore y^2 + B = \frac{A^2}{4}.$$

$\therefore$ When $\frac{A^2}{4}$ or $r = \text{min.}$, $y = 0$, $\therefore B = \frac{A^2}{4}$, or $\frac{a^2 b^2}{4} = \frac{A^2}{4}$,

and $a b = A = a b \cos C + r$, and

$$r = a b - a b \cos C = a b (1 - \cos C),$$

and $x^2 = \frac{A}{2} = \frac{a b \cos C + r}{2} = \frac{a b}{2}$, $\therefore x = \sqrt{\frac{a b}{2}}$, as before.

IV. *To find the least parabola which shall circumscribe a given circle.*

Since the parabola and the circle touch at E, therefore PE is a normal to the parabola, and PB is the subnormal = semi-parameter. Let $PB = z$; therefore the equation of the parabola is $y^2 = 2zx$. Also,

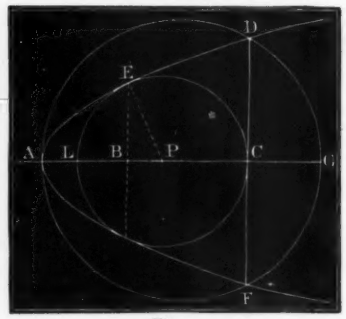


Fig.

$$EB^2 = r^2 - z^2, \therefore AB = \frac{r^2 - z^2}{2z}.$$

$$AC = AB + BP + PC$$

$$= \frac{r^2 - z^2}{2z} + z + r = \frac{(r+z)^2}{2z}.$$

Now the area of the parabola $FAD = \frac{4}{3} AC \times CF$ and $CF = \sqrt{2z \cdot AC}$.

$\therefore$ Area $FAD = \frac{4}{3} AC \times \sqrt{2z \cdot AC} = \frac{2(r+z)^3}{3z} = u = \text{min.}$

Let $r + z = y$; therefore $z = y - r$. Therefore $\frac{y^3}{y - r} = \text{min.} = u$, and therefore $y^3 - uy + ur = 0$. Let one of the negative roots of this equation $= -a$, and therefore $y + a$ must exactly divide the equation $y^3 - uy + ur = 0$.

$$\begin{array}{l|l} y + a & y^3 - uy + ur = 0 \quad (y^2 - ay + a^2 - u = 0 \ldots (B) \\ & \underline{y^3 + ay^2} \\ & \quad -ay^2 - uy \\ & \quad \underline{-ay^2 - a^2 y} \\ & \quad\quad (a^2 - u)y + ur \\ & \quad\quad \underline{(a^2 - u)y + a^3 - au} \\ & \quad\quad\quad ur - a^3 + au = 0; \end{array}$$

and therefore $a^3 - au = ru$; $\therefore u = \frac{a^3}{a+r}$.

Now solving the quadratic (B), we find

$$y = \frac{a}{2} \pm \sqrt{u - \frac{3a^2}{4}};$$

and in order that u may become a minimum, that is, the least value of u which will make y real, we must have

$$u = \frac{3a^3}{4},\ \therefore \frac{a^3}{a+r} = \frac{3a^2}{4},\ \text{or}\ \frac{a}{a+r} = \frac{3}{4},\ \therefore 3a + 3r = 4a;$$

$$\therefore \quad a = 3r,\ \therefore y = \frac{a}{2} = \frac{3r}{2}.\quad \therefore z = y - r = \frac{3r}{2} - r = \frac{r}{2}.$$

SECOND SOLUTION WITHOUT IMPOSSIBLE ROOTS.

In the equation $y^2 - ay + a^2 - u = 0$, or $y^2 - ay = u - a^2$, let $y = w + \frac{a}{2}$, and

$$\therefore\ y^2 - ay = w^2 + aw + \frac{a^2}{4} - aw - \frac{a^2}{2} = w^2 - \frac{a^2}{4} = u - a^2.$$

$\therefore u = w^2 + \frac{3a^2}{4}$; which is evidently a minimum, when $w^2 = 0$. $\therefore u = \frac{3a^2}{4}$; but $u = \frac{a^3}{a+r} = \frac{3a^2}{4}$, or $4a = 3a + 3r$, and $a = 3r$, and therefore $y = \frac{a}{2} = \frac{3r}{2}$ and $z = y - r = \frac{r}{2}$, as before.

The foregoing problems, extracted from a Treatise on Problems of Maxima and Minima solved by Algebra, must suffice as examples of the author's method. (See Notice.)

NOTE ON THE FORTY-SEVENTH PROPOSITION OF EUCLID.

BY JOHN M. RICHARDSON,
Professor of Mathematics, Collegiate Institute, Bowdon, Ga.

THE square described on the hypothenuse of a right-triangle is equal in area to the sum of the squares on the other two sides.

HISTORICAL ITEMS.

On account of its extensive application, this is one of the most interesting propositions in elementary plane geometry. It is variously denominated the Pythagorean Proposition, the Hecatomb Proposition, the Carpenter's Theorem, the Pons Asinorum of Mathematics. The name of its reputed discoverer, PYTHAGORUS, has given rise to the first; the sacrifice of a hundred fat oxen, which he made as a thank offering to the gods for the mental illumination that enabled him to discover it, to the second; its frequent application in building, to the third; and its supposed difficulty, to the fourth.

There seems to be some dispute, however, as to whether the fourth of the above names was given to the Vth, the XXth, or the XLVIIth proposition of EUCLID.

"PROCLUS in his 'Commentaries on EUCLID,'" says, "that the Epicureans derided the XXth proposition as being manifest 'even to asses;' for if a bundle of hay were placed at one extremity of the base of a triangle, and an ass at the other, the animal would not be *such an ass* as to take the crooked path to the hay instead of the straight one, as he would know the direct course to be the shorter: *this* was therefore called the 'asses' bridge.'"

CAMERER, in his notes on the first six books of EUCLID, collected *seventeen* demonstrations of this theorem.

HOFFMANN, in 1819, published a collection of *thirty-three* different solutions of it. I send you *twenty-eight* as the result of a short investigation. I have had no opportunity of comparing them with the other collections mentioned, but have given credit whenever I could; and with regard to the others, although they have resulted from my own hurried examinations, I cannot suppose that any of them are new. They have, doubtless, been discovered by different investigators over and over again. Many, however, may be new to the readers of the MONTHLY, and I send them as exercises for students.

Demonstration 1. Fig. 1. EUCLID.

Triangle $ABG = \frac{1}{2} ACGF = ACM = \frac{1}{2} AKML.$

$\therefore ACGF = AKML.$

Also, $BCDE = BKLH.$

$\therefore ACGF + BCDE = ABMH$

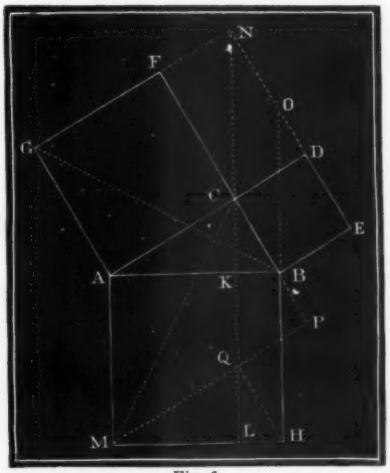


Fig. 1.

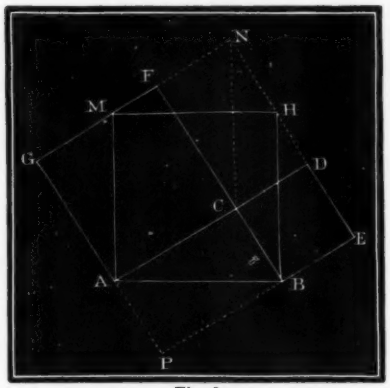


Fig. 2.

Dem. 2. Fig. 2. YOUNG.

$ABHM = ABENG - (BEH + HMN + MGA)$

$= ABENG - (ABC + CDNF)$

$= ACFG + CBED.$

But $ABC + CDNF = BEH + HNM + MGA.$

$\therefore \quad ABHM = ACFG + CBED.$

Dem. 3. Fig. 1. YOUNG. $LHBK = BONC = BCDE.$

Dem. 4. Fig. 3. LEGENDRE. From the similar triangles ABC, ACD, BCD, we have $AC^2 = AB.AD$, $BC^2 = AB.BD.$

$\therefore AC^2 + BC^2 = AB(AD + BD) = AB^2.$

Fig 3.

Dem. 5. Fig. 3. BEZOUT. OLIVER, Mathematical Monthly, Vol. I., p. 10. The triangles ABC, ACD, BCD, being similar, are to each other as the squares described upon their homologous sides, AB, AC, BC; but $ABC = ACD + BCD.$

$\therefore AB^2 = AC^2 + BC^2.$

Dem. 6. Fig. 2. KNAPSEN.

$$ABHM = PENG - (APB + BEH + HNM + MGA)$$
$$= PENG - (ACBP + CDNF) = ACFG + BCDE,$$

since $APB + BEH + HNM + MGA = ACBP + CDNF$.

Dem. 7. HOYT. See Mathematical Monthly, Vol. I., p. 231; also, p. 361.

Dem. 8. PAPPUS. YOUNG. See also Mathematical Monthly, Vol. I., p. 354.

Dem. 9. WHEELER. See Mathematical Monthly, Vol. I., p. 159.

Dem. 10. WHEELER. See Mathematical Monthly, Vol. I., p. 160.

Dem. 11. Fig. 4. $MRCA = MLKA = ACFG$;
$BCRH = BKLH = BCDE$.
$\therefore\ ABHM = ACFG + BCDE$.

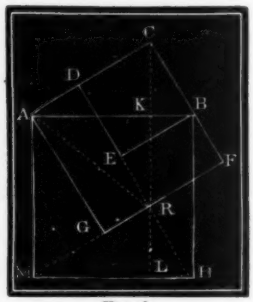


Fig. 4.

Dem. 12. Fig. 2. $ACNM + BCNH = ABHM$.
But $ACNM = ACFG$, and $BCNH = BCDE$.
$\therefore\ ABHM = ACFG + BCDE$.

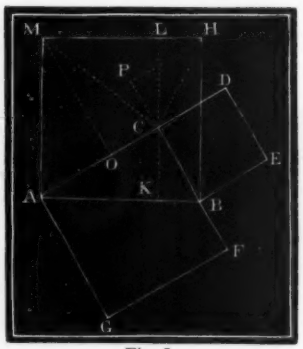


Fig 5.

Dem. 13. Fig. 5.
$ACM = \frac{1}{2} AKLM = \frac{1}{2} ACFG$,
and $BCH = \frac{1}{2} KBHL = \frac{1}{2} BCDE$.
$\therefore\ ABHM = ACFG + BCDE$.

Dem. 14. Fig. 6.
$ANC = \frac{1}{2} ACGH$
$= \frac{1}{2} AMLN$;

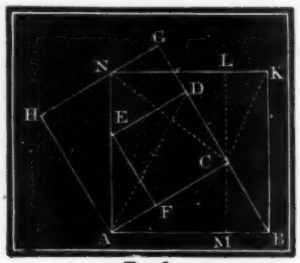


Fig. 6.

$AED = \frac{1}{2} CDEF = BCK = \frac{1}{2} MBLK$.
$\therefore\ ABKN = ACGH + CDEF$.

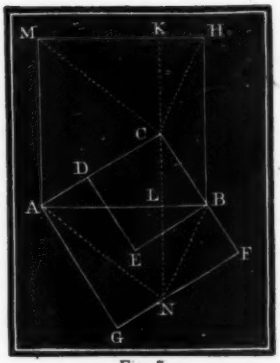


Fig 7.

Dem. 15. Fig. 7.

$ALKM = AMCN = 2\,ACN = ACFG;$

$BHCN = LBHK = 2\,BCN = BCDE.$

$\therefore ABHM = ACFG + BCDE$

Dem. 16. Fig 8.

$OCKG + CPEK = ONPC,$

$CDKF = BEPH = AGOM$

$= 2\,ABC = 2\,HMN.$

$\therefore ABHM = ACFG + BCDE$

Fig. 8.

Dem. 17. Fig. 1.

$ACQM = AKLM = ACFG;$

$BCQH = BKLH = BCDE.$

$\therefore ABHM = ACFG + BCDE.$

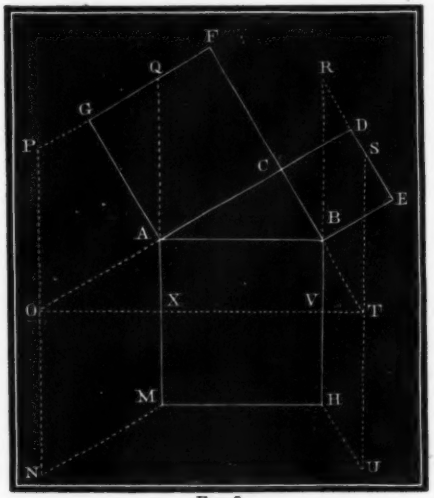


Fig 9.

Dem. 18. Fig. 8.

$ELGK = CONP;$

$CDKF = ACBL = 2\,ABC.$

$\therefore ABHM = ACFG + BCDE.$

Dem. 19. Fig. 9.

$AO = AC, BT = CB;$

$AOMN + BTUK = ABHM;$ but

$AOMN = AOPQ = ACFG,$

$BTUH = BTSR = BCDE.$

$\therefore ABHM = ACFG + BCDE.$

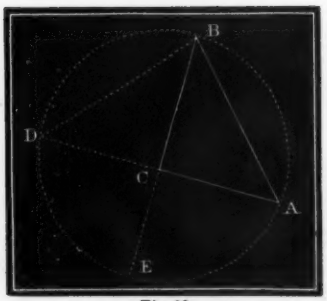


Fig 10.

Dem. 20. Fig. 10. YOUNG. This is an application of Prop. XXXII., Book IV. LEGENDRE.

$$CD = CA, \text{ and}$$
$$AB \times BD = BC^2 + AC \times CD.$$
$$\therefore \quad AB^2 = BC^2 + AC^2.$$

Dem. 21. Fig. 10. Application of Prop. XXXI., Book IV. LEGENDRE.

$$AB \times BD = BE \times BC = BC^2 + (BC \times CE = AC \times CD).$$
$$\therefore \quad AB^2 = BC^2 + AC^2$$

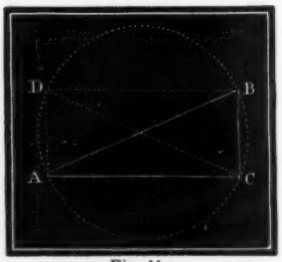


Fig. 11.

Dem. 22. Fig. 11. Application of Prop. XXXIII., Book IV. LEGENDRE.

$AB \times CD = AC \times BD + BC \times AD$, or
$$AB^2 = AC^2 + BC^2.$$

Dem. 23. Fig. 3. Take $AE = 1$,

$$CD = \frac{AD \times FE}{AF}, \; BD = \frac{CD \times FE}{AF} = AD \times \frac{FE^2}{AF^2}.$$
$$\therefore \; AB = AD + BD = AD + AD \times \frac{FE^2}{AF^2} = AD\left(1 + \frac{FE^2}{AF^2}\right);$$

or $AB = AD\left(\frac{AF^2 + FE^2}{AF^2}\right)$. But $AB = \frac{AC}{AF}$, and $AC = \frac{AD}{AF}$; hence, $AB = \frac{AD}{AF^2}$. Comparing these two values of AB, we get $AF^2 + FE^2 = 1$. Again, $AB \times AF = AC$, and $AB \times FE = BC$. Squaring these and adding

$$AB^2(AF^2 + FE^2) = AB^2 = AC^2 + BC^2.$$

Dem. 24. Fig. 3. Take $AG = 1$. If AB^2 is not equal to $AC^2 + BC^2$, let $x^2 = AC^2 + BC^2$; then

$$x = (AC^2 + BC^2)^{\frac{1}{2}} = AC\left(1 + \frac{BC^2}{AC^2}\right)^{\frac{1}{2}} = AC\left(1 + \frac{EG^2}{AE^2}\right)^{\frac{1}{2}}$$
$$= AC\left(\frac{AE^2 + EG^2}{AE^2}\right)^{\frac{1}{2}} = \frac{AC}{AE}$$

by Dem. 23. Therefore $AB^2 = x^2 = AC^2 + BC$.

Dem. 25. Fig. 12. Application of Prop. XXIX., Book IV. LEGENDRE. $AE \times AB = AG \times AD$,

or $(AB - 2BF)AB$

$= (AC + BC)(AC - BC)$

$\therefore AB^2 = AC^2 - BC^2 + 2BF.AB$

$= AC^2 + BC^2$.

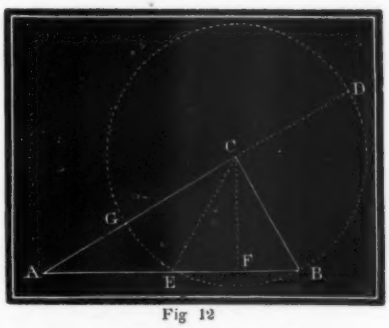


Fig 12

Dem. 26. Fig. 13. Application of Prop. XXX., Book IV. LEGENDRE.

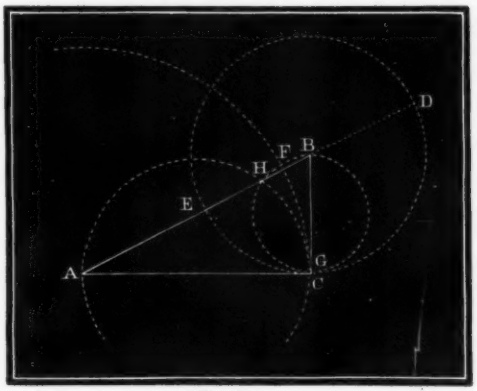


Fig 13.

$AC^2 = AE \times AD$

$= AE \times AB + AE \times BC$.

$AC^2 + BC^2 = AE \times AB$

$+ AE \times BC + BC^2$

$= AE \times AB + BC \times AB$

$= AB(AE + BC) = AB^2$; or

$AC^2 = AE \times AD$

$= (AB - BC)(AB + BC)$

$= AB^2 - BC^2$.

$\therefore AB^2 = AC^2 + BC^2$.

Dem. 27. Fig. 13. Application of the same proposition.

$$AC^2 = AH \times AB,\ BC^2 = BH \times AB.$$

Adding, we get $AC^2 + BC^2 = AH \times AB + BH \times AB = AB^2$.

Dem. 28. Fig. 13. Application of the same proposition.

$$AC^2 = AE(AB + BC),\ BC^2 = BF(AB + AC).$$

NOTE. Several of these demonstrations had already been received in nearly the same form; Dem. 11, from PLINY EARLE CHASE, Philadelphia; Dem. 22, from C. M. RUNK, Allentown, Pa.; Dem. page 361, Vol. I., JOSEPH FICKLIN, Jr., Trenton, Mo.; Dem. 26, second form, C. J. KEMPER, Harrisonburg, Va., and Prof. CHARLES A. YOUNG, Hudson, Ohio. The student will remember that the propositions upon which the Pythagorean is made to depend, must be proved independently of the latter.

Adding we get

$$\begin{aligned}AC^2+BC^2&=AE(AB+BC)+BF(AB+AC)\\&=AE\times AB+AE\times BC+BF\times AB+BF\times AC\\&=AB(AE+BF)+AE\times BC+BF\times AC+BF\times AB-BF\times AB\\&=AB(AE+BF)+AE\times BC+BF(BF+2AC)-BF\times AB\\&=AB(AE+BF)+AE\times BE+BE^2-BF(AE+EB)\\&=AB(AE+BF)+(AE+EB)(BE-BF)\\&=AB(AE+BF)+AB\times EF=AB(AE+EF+FB)=AB^2.\end{aligned}$$

NOTE "ON THE HORIZONTAL THRUST OF EMBANKMENTS."

BY F. W. BARDWELL,
Professor of Mathematics in Antioch College, Yellow Springs, Ohio.

In an article "On the Horizontal Thrust of Embankments," in Vol. I., p. 175, I find what seems to me to be an error, which makes the principal formula defective, and therefore affects the conclusion. Without repeating the preliminary explanation, I will simply quote the portion which seems to be wanting in accuracy.

"The variable forces, F' and Q, may be resolved into the components $F'\cos v$, $Q\sin v$, perpendicular to af, and $F'\sin v$, $Q\cos v$, parallel to af. The force, $F\sin v$, acting along af upwards, must, assisted by the friction due to the normal pressure on af, just equal the parallel and opposite force $Q\cos v$; that is,

(1) $\qquad F'\sin v+(F'\cos v+Q\sin v)f=Q\cos v.$"

But the normal pressure is $Q\sin v$ only, and equation (1) should be

$$F'\sin v+Qf\sin v=Q\cos v.$$

This corresponds to the formula usually given. I will only add that no allowance is made in the article referred to for the cohesion of particles.

NOTE ON DOUBLE POSITION.

BY REV. THOMAS HILL,
President of Antioch College, Yellow Springs, Ohio.

In the Cambridge Miscellany, Prof. Peirce expressed his regret that the old "Rule of False" had been omitted from the more recent elementary treatises on Arithmetic, published in this country. Since the date of that note, several Arithmetics have been published, containing the rule,— introduced however, as it were, timidly, and in a corner of the book. Yet the fundamental idea of the rule of double position lies at the basis of every attempt of the human intellect to discover truth. We frame an hypothesis, compare it with facts, and note the amount of discrepancy; then, for the corrections of our hypothesis, assume that the errors of our results are in some measure proportional to the errors of our data.

Now in mathematics, this gives a practical mode of attaining numerical results. Thus, let x be an unknown number, required to produce a fixed result, a. Let x' and x'' be two supposed values of x, which lead to the fixed results, $a+e$, and $a+e'$. Now, according to the general assumption of proportionality of errors in results to those in the data,

$$e : e' = x' - x : x'' - x.$$

And this leads to the value of x,

$$x = \frac{e\,x'' - e'\,x'}{e - e'}.$$

The identity of this formula with the following rule, given in Daboll's Arithmetic, is manifest.

"Multiply the first position by the last error, and the last position by the first error. If the errors are alike, [that is, both results too great, or both results too small,] divide the difference of the products by the difference of the errors, and the quotient will be

the answer. If the errors are unlike, divide the sum of the products by the sum of the errors, &c."

The advantage of this form of the rule is the ease with which it disposes of the signs of the errors. If the problem is an equation of the first degree, the answer is at once obtained with perfect accuracy; if not, the process must be repeated. For example: What is the side of a cube when the inches of superfices exceed the inches of solidity by 20?

Suppose a side of 2 inches. Then $6(2)^2 - 2^3 = 16$ and $20 - 16 = 4$. Suppose a side of 3 inches. Then $6(3)^2 - 3^3 = 27$ and $20 - 27 = -7$. Hence, by the rule, the approximate side is $(2 \times 7 + 3 \times 4) \div (7 + 4) = 2.36$.

Taking, therefore, the two positions, 2.3 and 2.4, we have $6(2.4)^2 - (2.4)^3 = 20.74$ and $20 - 20.74 = -.74$,

$$6(2.3)^2 - (2.3)^3 = 19.57 \text{ and } 20 - 19.57 = .43.$$

Then by the rule

$$[(.74 \times 2.3) + (.43 \times 2.4)] \div (.43 + .74) = 2.336 +.$$

Taking the new positions 2.336 and 2.337,

$6(2.336)^2 - (2.336)^3 = 19.994$; and $20 - 18.994 = .006$

$6(2.337)^2 - (2.337)^3 = 20.0058$; and $20 - 20.0058 = -.0058$.

Hence, for a nearer approximation, the side equals

$$[(.006 \times 2.337) + (.0058 \times 2.336)] \div (.006 + .0058) = 2.336508.$$

A simple rule, but requiring more care in the signs, consists in making the correction of the first position the unknown quantity. By the doctrine of proportions we readily obtain, from the given proportion, the following value of this correction,

$$x' - x = \frac{e(x' - x'')}{e - e'}.$$

That is to say, *Multiply the error of the first result by the difference of*

the positions, and divide by the difference of the results; the quotient is a correction to be added to the first position, if the first result shows it to be too small, and to be subtracted from the first position if the first result shows it to be too large.

This rule we prefer to the old rule quoted from DABOLL, although it gives, of course, the same result.

ON THE INDETERMINATE ANALYSIS.

By Rev. A. D. WHEELER, Brunswick, Maine.

[Continued from Page 25.]

PROPOSITION VI. If a and b be prime to each other, the indeterminate equation, $ax - by = c$, is always possible; and will admit of an infinite number of positive integral solutions.

DEMONSTRATION. Transferring and dividing, we have $x = \frac{c+by}{a}$, which has already been shown to be possible. PROP. V., Cor. 2.

Now as the remainders, resulting from the division of $c + by$ by a, recur in periods, and the number of periods is unlimited, since y may have all possible values, it follows that the number of solutions must also be unlimited; that is, infinite.

PROP. VII. If, in the equation $ax \pm by = c$, we call the least integral value of x, v; the next greater value of x, when the equation admits of more than one solution, will be $v + b$, the next $v + 2b$, and so on in Arithmetical progression.

DEM. The least value of x being v, and d denoting the difference, we shall have $v + d$ for the next greatest. Then $ax \pm by$ becomes $av \pm by' = c$ in the first case, or $y' = \frac{c - av}{\pm b} =$ an integer. In the second case, it becomes $av + ad \pm by'' = c$, or $y'' = \frac{c - av - ad}{\pm b}$ $=$ an integer. Therefore $y'' - y' = \frac{c - av - ad}{\pm b} - \frac{c - av}{\pm b} = \frac{-ad}{\pm b}$.

(Ax. 2). Therefore $\frac{d}{b}$ is an integer. (Prop. III.) But this can be an integer only when $d = b$, $d = 2b$, &c., which was to be proved.

Prop. VIII. The equation $ax + by = c$ is always possible for n solutions, when $c > n\,a\,b$.

Dem. Let $c = nab + r$. Then we have

$$ax + by = nab + r\text{; or}$$
$$ax - nab = -by + r\text{; or}$$
$$x - nb = \frac{-by + r}{a} = -\frac{by - r}{a} = \text{an integer,}$$

(Prop. V. Cor. 1). Therefore the equation admits of at least one solution.

Let $x = v$ for its first value; $x = v + b$ for its second; $x = v + 2b$ for its third; and so on, (Prop. VII). Then we shall have $x = v + (n-1)\,b$, for its nth value; and substituting this for x, we have

$$av + (n-1)\,ab + by = nab + r\text{; or}$$
$$av + by = ab + r\text{; or}$$
$$av - ab = -by + r\text{; or}$$
$$v - b = -\frac{by - r}{a}\text{, the same as before.}$$

Therefore the equation $ax + by = c$, will always admit of n solutions when $c > nab$.

Prop. IX. The equation $ax + by = c$ is impossible, in positive whole numbers, in the following cases. (1.) When a, or b, is prime to c, but not prime to each other. (2.) When $c < a + b$. (3.) When $c = ab$. (4.) When $c = ab - (ax' + by')$.

Dem. *Case* 1. Since a and b are supposed to have a common factor which is not in c, it is obvious that one member of the equation is divisible by it, while the other is not.

Case 2. The smallest integral value which can be given to x or y is 1. Giving to them this value, the equation becomes $a + b = c$. Consequently if c is less than $a + b$, the solution is impossible.

Case 3. Let $c = ab$; then we have

$$ax + by = ab$$

Now as a will divide one member of the equation, it must also divide the other, (PROP. I.); and consequently must divide by. But it cannot divide b, because it is prime to it. Therefore it must divide y, (PROP. III). Hence, $y = a$ is its least integral value. Substituting this value in the place of y, we have $ax + ab = ab$, or $ax = 0$. Wherefore $x = 0$; and the conditions of the equation cannot be fulfilled.

Case 4. Let $c = ab - (ax' + by')$.

Then we have $ax + by = ab - (ax' + by')$, or transposing,

$$a(x + x') + b(y + y') = ab.$$

Whence as before, $y + y$ will equal a, for its least value; and $x + x'$ will become zero. This case is therefore proved.

(To be continued.)

COMPLETE LIST OF DR. BOWDITCH'S WRITINGS.

To the various volumes of the Transactions of the American Academy of Arts and Sciences, Dr. BOWDITCH communicated the following memoirs:

Vol. II., Part II., Published in 1800.

1. *A New Method of Working a Lunar Observation.*

The object of this method was to establish a uniform rule for the application of corrections, so that there should be no variation of cases resulting from the distance and altitude of the observed bodies.

Dr. BOWDITCH says of this method, in a note, that "it was written several years ago, and before the publication of the Transactions of the Royal Society for 1797, in which is inserted a method, somewhat similar, invented by Mr. MENDOZA Y RIOS. An appendix to the New Practical Navigator has lately been published, in which the corrections are all additive, and the work is shorter." It is particularly noticed and commended in the Connoissance des Tems (1808) then published under the direction of M. DELAMBRE.*

* ZACH (Corr. Astron. Vol. VI., p. 553, A. D. 1822), says: "M. BOWDITCH dans son *New American Practical Navigator* a aussi donné pour la réduction des distances lunaires une nouvelle méthode

Vol. III., Part I., Published in 1809.

2. *Observations on the Comet of* 1807. [pp. 1–18.]

3. *Observations on the Total Eclipse of the Sun, June* 16, 1806, *made at Salem.* [pp. 18–23.]

In a note to this communication, Dr. BOWDITCH makes, as is believed, the first public mention of an error in LAPLACE'S *Mécanique Céleste*, in the estimate of the oblateness of the earth, as calculated from the length of pendulums; showing that LAPLACE'S result ought to have been, upon his own principles, $\frac{1}{315}$ instead of $\frac{1}{326}$.

4. *Addition to the Memoir on the Solar Eclipse of June* 16, 1806. [pp. 23–33].

5. *Application of* NAPIER'S *Rules for Solving the Cases of Right-Angled Spheric Trigonometry to several Cases of Oblique-Angled Spheric Trigonometry.* [pp. 33–38].

This communication so alters NAPIER'S rules, as to make them include most of the cases of oblique-angled spheric trigonometry, and is marked by the same neatness, elegance, and simplicity, which characterized his first communication. These rules are now familiarly known in the text-books of Harvard College as "BOWDITCH'S Rules."

Vol. III., Part II., Published in 1815.

6. *An Estimate of the Height, Direction, Velocity, and Magnitude of the Meteor that exploded over Weston, in Connecticut, December* 14, 1807. [pp. 213–237].

This communication is of a very interesting character, and it rests upon numerous observations collected with great labor and assiduity. Dr. BOWDITCH considers the meteor in question to have had a course about eighteen miles above the earth, a velocity of more than three miles a second, and a probable cubic bulk of six millions of tons, — which others have estimated to be the contents of the pyramid of Cheops.*

7. *On the Eclipse of the Sun of September* 17, 1811, *with the Longitudes of several Places in this Country, deduced from all the Observations of the Eclipses of the Sun, and Transits of Mercury, and Venus, that have been published in the Transactions of the Royal Societies of Paris and London, and the Philosophical Society held at Philadelphia, and the American Academy of Arts and Sciences.*† [pp. 255–305.]

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abrégée, avec des tables, qui mérite d'être plus connue; aucun auteur Européen n'en a encore parlé; il vient de la perfectionner dans sa quatrième édition stéréotpye publiée à New York en août 1817. Nous la recommandons à l'attention des professeurs et auteurs des traités de navigation." In Vol. X., p. 321, A. D. 1824, he says: "La méthode de M. BOWDITCH a l'avantage sur toutes les autres méthodes d' approximation, que toutes les corrections sont toujours additives, et qu'on n'a jamais besoin de faire attention à des cas particuliers; les règles sont générales;" and proceeds to give a detailed account of it. — See also note to article 15.

* The *Zeitschrift für Astronomie*, Vol. I., p. 37, A. D. 1816, gives the results arrived at in this communication, and calls it "einer interressanten Arbiet."

† The *Zeitschrift für Astronomie*, Vol. I., p. 90, 1816, mentions the observations of the eclipses of the sun, June 16, 1806, and September 17, 1811, as contained in these volumes, &c., and states that "BOWDITCH hat den grössern Theil davon zu Längenbestimmungen benutzt und zugleich dabey, für eine Menge Amerikanischer Orte, Hülfsgrössen zur leichtern Berechnung des Nonagesimus gegeben;"

8. *Elements of the Orbit of the Comet of* 1811. [pp. 313—326].

In this, as in his second communication, he arrives at his results after almost incredible labor, rendered necessary by the want of the improved methods of the present day.* The original volume, containing his calculations in the case of this latter comet, now preserved in his library, contains one hundred and forty-four pages of close figures, probably exceeding one million in number, though the result of this vast labor forms but a communication of twelve pages.†

9. *An Estimate of the Height of the White Hills in New Hampshire.* [pp. 326—328].

10. *On the Variation of the Magnetic Needle.* [pp. 337—344].

This communication, in like manner, which is of quite an interesting character, and of considerable practical importance, was the result of five thousand and twenty-five observations, during a period of five years.

11. *On the Motions of a Pendulum suspended from two points.* [pp. 413—437].

This communication is also one of interest and value; and the little wooden stand, from which a leaden ball was suspended, still exists, to remind us of the zeal and assiduity with which Dr. BOWDITCH watched the various curves and lines which the ball described.‡

12. *A Demonstration of the Rule for finding the Place of a Meteor, in the Second Problem, page* 218 *of this Volume.* [pp. 437—439]

Vol. IV., Part I., Published in 1818.

13. *On a Mistake which exists in the Solar Tables of Mayer, Lalande, and Zach.*§

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and ZACH, in his Corr. Astron. Vol. X., p. 494, A. D. 1824, has a table of the longitudes and latitudes of places determined by astronomical observations calculated by Dr. BOWDITCH.

* See Dr. BOWDITCH's letter (ZACH, Corr. Astron. Vol. X., p. 228), before referred to, where this fact is stated. The editor, in page 248, gives the elements of the orbits of the comets calculated by Dr. BOWDITCH wholly from American Observations.

† Mr. ENCKE, in speaking to a friend of Dr. BOWDITCH, at Berlin, in 1836, said that he had known him from the time when this paper appeared; and that he had never seen an American since, without asking him what he could tell him about its author; and the *Zeitschrift für Astronomie,* Vol. I., p. 44, gives an account of this communication "von dem Amerikanischen Astronomen BOWDITCH."

‡ This subject is mentioned is his letter to Baron ZACH, before alluded to, (*Corr. Astron.* Vol. X., p. 227). The editor, in his note, p. 246, says the remarkable variety of the motions of a pendulum thus suspended, and the very curious experiments of Prof. DEAN, who explains, in this mode, the apparent motion of the earth as seen from the moon, engaged Dr. BOWDITCH in the examination of the theory of these motions. The result has been, he adds, "une recherche très intéressante." "Comme ce mémoire mérite d'être mieux connu, et qu'il ne l'est pas généralement, vu la difficulté de se procurer des livres Américains, nous en donnerons la traduction dans un de nos cahiers."

§ Dr. BOWDITCH states, that "The attraction of Jupiter produces an equation in the expression of the Sun's distance from the earth, and a Table is given for its computation, by MAYER, in 1770," &c., "and ever since this table was first published, which is about fifty years, an error of six signs has always existed in the argument, by which the correction is found; so that, when the equation is really *subtractive,* it will frequently be found by the table to be *additive,* and the contrary." "In DE LAM-

14. *On the Calculation of the Oblateness of the Earth, by means of the observed Lengths of a Pendulum in different Latitudes, according to the Method given by Laplace, in the Second Volume of his "Mécanique Céleste;" with remarks on other parts of the same Work relating to the Figure of the Earth.* [pp. 3—24.]

The object of this communication is to correct certain errors in the article "Earth," in REE'S Cyclopædia, to the end that currency should not be given to inaccurate ideas on the subject, by that popular work.

15. *Method of Correcting the apparent Distance of the Moon from the Sun, or a Star, for the Effects of Parallax, and Refraction.* [pp. 24–31.]

This is but the rule given in the Practical Navigator, making all the corrections in question additive. It is another instance of the simplicity at which he always aimed in his rules and formulas.*

16. *On the Method of computing the dip of the Magnetic Needle in different Latitudes, according to the Theory of Mr. Biot.* [pp. 31—36.]

17. *Remarks on the Methods of correcting the Elements of the Orbit of a Comet, in Newton's "Principia" and in Laplace's "Mécanique Céleste.* [pp. 36—48.]

This communication proves that two equations in the Principia, the accuracy of which several commentators upon that work had attempted to prove, and as to which no doubts had yet been expressed or insinuated, always made the corrections in question "double of what they ought to be," and restricts the method of LAPLACE as appropriate only where the number of observations is small.

18. *Remarks on the usual Demonstration of the Permanency of the Solar System, with respect to the Eccentricities and Inclinations of the Orbits of the Planets.* [pp. 48—51.]

19. *Remarks on Dr. Stewart's Formula for computing the Motion of the Moon's Apsides, as given in the Supplement to the Encyclopædia Britannica.* [pp. 51—61.]

This is a very curious and interesting communication. A method which, notwithstanding doubts had been expressed respecting it, had been sanctioned as accurate by

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BRE'S Solar Tables, published in 1806, the form of the table is wholly altered, the method of entry by a double argument being used; and by thus taking a different path, the error is avoided, without noticing that it really does exist in the other works."

Baron ZACH, in his *Monatliche Correspondenz*, Vol. VIII., p. 449, A. D. 1803, says that BOWDITCH, an American astronomer, has called attention to this mistake; and, after admitting its importance, frankly adds, "Allen Astronomen, welche sich mit Verfertigung der Sonnen-Tafeln beschäftigt haben, einen LA CAILLE, TOB. MAYER, LA LANDE, DE LAMBRE und *mir* ist dieser Fehler entgangen."

* In ZACH'S *Monatl. Corres.*, Vol. XVII., p. 411, A. D., 1808, this method is mentioned as being in the Appendix to the New American Practical Navigator, printed in Newburyport, 1804; and the editor says: "Der Verfasser ist ein Americaner, BOWDITCH, und DE LAMBRE hat es der Mühe werth gehalten eine umständliche Darstellung dieses Verfahrens zu geben." Then follows a somewhat minute account of the method. See note to Article I.

Dr. HUTTON, by LALANDE, and PLAYFAIR,—the latter of whom even considered its accuracy to have been demonstrated,—is in this memoir proved to have been true only in the particular case supposed; and it is shown that, as a general method, it wholly fails.

Vol. IV., Part II., Published in 1820.

20. *On the Meteor which passed over Wilmington, in the State of Delaware, November* 21, 1819. [pp. 3—14.]

21. *Occultation of Spica by the Moon, observed at Salem.* [p. 14.]

22. *On a Mistake which occurs in the Calculation of Mr. Poisson relative to the Distribution of the Electrical Matter upon the Surfaces of two Globes, in Vol. XII., of the Mémoires de la classe des sciences mathématiques et physiques de l' Institut Impérial de France.* [pp. 15—17.]

23. **Elements of the Comet of* 1819. [pp. 17—19.]

Besides the above contributions to the Memoirs of the American Academy, Dr. BOWDITCH was the writer of several other articles, among which may be mentioned the following:

1. *Notice of the Comet of* 1807. Published in the Monthly Anthology, for December, 1807, Vol. IV. [pp. 653—654.]

2. *Review of a "Report of the Committee (of Congress) to whom was referred, on the 25th January,* 1810, *the Memorial of William Lambert, accompanied with sundry Papers relating to the establishment of a First Meridian for the United States, at the permanent seat of their Government.* Published in the Monthly Anthology, for October, 1810, Vol. IX. [pp. 245—266.]

This article occupies twenty-one pages, and proves very conclusively the great advantages of continuing to estimate the longitude from Greenwich, which Mr. LAMBERT considered "a sort of degrading and unnecessary dependence on a foreign nation," and "an incumbrance unworthy of the freedom and sovereignty of the American people." This Memorial the reviewer shows to be a compilation, with needless repetitions and palpable mistakes, evincing a great want of knowledge in the principles of the calculations;" and that, "both as respects its object and execution, it was wholly undeserving the patronage of the National Legislature."

3. *Defence of the Review of Mr. Lambert's Memorial.* Published in the Monthly Anthology, for January, 1811, Vol. X. [pp. 40—49.]

Mr. LAMBERT having made an angry reply, charging his reviewer with "twistical cunning," "ingenious quibbling," "zeal for the honor of the British nation, and convenience of British mariners," and challenging him "to examine his computation of the longitude of the Capitol at Washington from Greenwich, *and to point out a mistake that*

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* For a statement of Dr. BOWDITCH's communications to the Memoirs of the Academy, and an abstract of their contents, from which several of our remarks in the text are condensed, see Mr. PICKERING's Eulogy, [pp. 17–31].

can be made palpable." — Dr. BOWDITCH, in his reply, considers these charges of Mr. LAMBERT as beneath his notice, but accepts his challenge, and proves *that there is an error in every one of the six examples he has given.*

These two papers were fatal to the proposed project; and fortunately for the interests of science, Greenwich continues to be the first meridian of all who speak the English language.

4. *Review of a Treatise on the most easy and convenient Method of computing the Path of a Comet, from several Observations; by William Olbers, M. D., Weimar*, 1797;" *and of "Theoria Motus Corporum Cœlestium in Sectionibus Conicis Solem ambientium;"* by Charles Frederick Gauss; Hamburg, 1809. Published in the North American Review, for April, 1820, Vol. X. [pp. 260—272.]*

This article gives an account of several German astronomers and their most noted periodical publications. Thus it contains a notice of Dr. OLBERS — "the Columbus of the planetary world," — and of GAUSS, the authors of the two works reviewed; an account of *Bode's Astronomisches Jahrbuch, Zach's Monatliche Correspondenz* and the *Zeitschrift für Astronomie.* It states the fact that, "out of *thirteen* primary planets and satellites, discovered since the year 1781, we are indebted to persons born in Germany for *twelve;* and that, in the determination of the orbits of these new bodies, they have done more than all the other astronomers in the world."

5. *Review of "A remarkable astronomical Discovery, and Observations of the Comet of July*, 1819; *by Dr. Olber's, of Bremen; published in Bode's Astronomisches Jahrbuch for* 1822; [and of two other articles in the same work, for the years 1822 and 1823, on the same subject, by Professor ENCKE of the Ducal Observatory at Seeberg, near Gotha.] Published in the North American Review, for January, 1822, Vol. XIV. [pp. 26—34.]

A copy of this article on ENCKE's Comet Dr. BOWDITCH also sent with the letter before mentioned to Baron ZACH, who, in his notes, states that Dr. BOWDITCH, "has here collected all that has been said and done respecting this famous comet." In the concluding paragraph of this article, the reviewer expresses his regret that, "while Great Britain alone can boast of more than thirty public and private observatories

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* A copy of this article Dr. BOWDITCH sent to Baron ZACH, with the letter before referred to, *marking a part of it as written by Mr. Everett, the editor.* ZACH publishes extracts from it in his notes upon this letter, Vol. X., p. 231, and says: "It will be interesting to the reader to learn how men of science in America render justice to those of Germany, while they reproach their bretheren beyond the water for the little attention which they have bestowed upon our productions." Dr. BOWDITCH mentions in this review an interesting paper which Mr. IVORY had published in the Transactions of the Royal Society of London, 1814, giving a method of his own for computing the orbit of a comet, which "upon examination turns out to be nothing more than that which Mr. OLBERS had published in his work above seventeen years before, although this coincidence must have been wholly unknown to Mr. IVORY, and to the other members of the Royal Society. We consider this as a striking instance of the little attention paid in Great Britain to works of mathematical science printed in Germany." The passage added by Mr. EVERETT was merely that which states a like neglect of German *literature.*

of considerable note, we have not, in the whole United States, one that deserves the name." He also speaks of the duties imposed on the importation of mathematical instruments and scientific works, as *fines* and *penalties*, which had been justly called "a bounty on ignorance," &c. This whole paragraph is extracted by ZACH, (Vol. X., p. 245), and he says, "Voici de quelle maniere un bon républicain exhales on chagrin en public; c'est au moins quelque chose, &c."

6. *Letter to Baron Zach, dated November* 22, 1822; *with a Postscript, dated December* 20, 1823. Published in his Correspondance Astronomique for the year 1824.* Vol. X. [pp. 223—230.]

7. *Review, entitled "Remarks on several Papers published in former volumes of this Journal;"* [the first being remarks on "A new Algebraic Series, by Professor WALLACE, of Columbia, S. C.] published in Silliman's Journal for 1824, Vol. VIII. [pp. 131—139] — and *Remarks on Mr. Wallace's Reply*, published in the same Journal for 1825, Vol. IX. [pp. 293—304.]

The reviewer expresses his surprise that any offence should have been given by the mere statement of the *historical fact* that this "new series" was but the usual development of the binomial theorem, and the same which had been given by EULER fifty years before.†

8. *Review of "Fundamenta Astronomiæ,"* by FREDERICK WILLIAM BESSEL; 1818; of the tables of the Moon, by M. BUCKHARDT; 1812; — of the New Tables of

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* This letter has been already more than once referred to, and contains many interesting facts. The editor's comments upon it occupy twenty pages. With this letter Dr. BOWDITCH had sent, besides his articles mentioned in the two last preceding items, a copy of the fifth edition of the Practical Navigator. The editor says of him: "C'est le premier, et jusqu' à-present le seul grand géomètre en Amérique."

† Professor WALLACE in his reply, states that he did not claim the series as *new*, and appeals to a reference which he had made in his original article to Mr. STAINVILLE, &c., and not knowing who his opponent was, says that he does not, like his reviewer, refer his readers to the Complement des Eléméns d'Algèbre, *however useful as a school book*, &c. He also states, "that the results which EULER has given do not include a single case of a transcendent function, and were only given as examples of the applications of the simplest case of the binominal theorem," &c. Dr. BOWDITCH in his rejoinder mentions the vague terms in which Mr. STAINVILLE had been originally referred to, and says, "It now appears that Mr. STAINVILLE gave it as new for the *first* time in 1818, and Professor WALLACE for the *second* time in 1824, EULER's having been published in 1775;" and again: "It is believed that most persons, after reading what Professor WALLACE has written, would suppose he claimed some, if not a very large portion, for his own. But the real fact is, that *none* of it is *his*. The whole of the first seven pages, and a large portion of the two remaining pages, of Professor WALLACE's first communication, are merely literal translations from STAINVILLE and GERGONNE; and what is not copied from them is quite unimportant." He also says: "It is a fact, *notwithstanding the positive* declaration of Professor WALLACE to the contrary, that EULER's demonstration is not restricted to this very simple case, but is general for all values of the exponent, whether integer, fractional, negative, or surd; and it is characterized by LACROIX as being elegant and rigorous." This review will be found quite amusing and piquant. It is, like the articles on Mr. LAMBERT's Memorial, both as to matter and style, a fair specimen of Dr. BOWDITCH's powers as a controversial writer.

Jupiter and of Saturn, by M. BOUVARD; 1808;—of the Tables of the Satellites of Jupiter, &c., by M. DELAMBRE; 1817;—of the Tables of Venus, of Mars, and of Mercury, by B. DE LINDENAU; 1810, 1811 and 1813;—and of the Memoir on the Figure of the Earth, by M. DE LAPLACE; 1817 and 1818. *Published in the North American Review for April 1825, Vol. XX. [pp. 309—367.]†

This brief but most comprehensive article upon modern astronomy will be found to possess an uncommon degree of interest. It consists of a series of biographical sketches, in which are described all who have been remarkable for the successful cultivation of physical science in modern times, bringing into view their actual and relative services and merits, and awarding to each the degree of approval to which he was entitled;—the writer now dwelling with enthusiasm upon his favorite LAGRANGE, now bestowing a more qualified and guarded approbation, or a positive censure upon others inferior in powers and attainments to that distinguished mathematician or opposite to him in character.‡ It comprises especially a very full account of Dr. BRADLEY's observations, and of BESSEL's services in reducing them; of the best makers of mathematical instruments—GRAHAM, BIRD, RAMSDEN, TROUGHTON, JONES, REICHENBACH, FRAUENHOFER, HERSCHEL, &c., of the successive astronomers royal at Greenwich, and of the other chief European observers; and lastly, "it gives an account of the labors of those mathematicians who have improved the science of astronomy by their calculations of the effects of the mutual attractions of the heavenly bodies."

Mécanique Céleste, by the Marquis DE LAPLACE, Peer of France, Grand Cross of the Legion of Honor; Member of the French Academy; of the Academy of Sciences of Paris, of the Board of Longitude of France, of the Royal Societies of London and Göttingen, of the Academies of Sciences of Russia, Denmark, Sweden, Prussia, Holland, and Italy; Member of the American Academy ot Arts and Sciences, &c. Translated with a Commentary, by NATHANIEL BOWDITCH, LL. D., Fellow of the Royal Societies of London, Edinburgh, and Dublin; of the Astronomical Society of London, of the Philosophical Society held at Philadelphia, of the

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* The titles of the particular works reviewed are here given in an abridged form.

† In the notes to Mr. PICKERING's Eulogy, p. 95, a list is given, without comment, of six of the above eight articles, the fifth and seventh not being noticed. All these occasional publications of Dr. BOWDITCH, except the letter to Baron ZACH, were collected by him in two volumes, now in his library.

‡ Thus, he says, "Upon the decease of EULER, LAGRANGE remained indisputably the greatest mathematician then living," &c.; while of Dr. BRADLEY's successor he says: "Dr. BLISS was wholly unworthy of the office of Astronomer Royal. The account of his life by LALANDE, is comprised in less than a dozen words — BLISS était Astronome Royal: il mourut en 1765." This article is the one, of all Dr. BOWDITCH's occasional publications, which exhibits in the clearest light, his peculiar talents and acquirements. Evidently the work of one possessing a knowledge of the actual state of mathematical science, in its various departments, as extensive and minute as was possessed by any individual then living,— it is throughout a record of the most sound and impartial criticism. Any biography of him which has not this review in an appendix, must be incomplete.

American Academy of Arts and Sciences; Corresponding Member of the Royal Societies of Berlin, Palermo, etc. Vol. I., pp. 746, 1829. Vol. II., pp. 990, 1832. Vol. III., pp. 1017, 1834. Vol. IV., pp. 1018, 1839. Boston: From the press of Isaac R. Butts: Charles C. Little and James Brown, Publishers.

Dr. Bowditch was also, for many years, a contributor to the "Annalist" and "Mathematical Diary," solving every question there proposed, in his usual style of simple elegance. He also wrote or corrected various articles in the American edition of Rees's Cyclopædia. And all these various publications were the employment merely of those leisure hours which were left to him after all the calls of active business, and all the claims of social and domestic life, had been most fully answered and more than this, notwithstanding all the occasional scientific labors which have been mentioned, such was his wonderful economy of time, that, within the same period, he also completed what has justly been characterized as the gigantic undertaking of making the Translation and Commentary of *Mécanique Céleste*, a work upon which, almost exclusively, will rest his fame as a man of science.* Upon recurring to the Translator's Preface in the first volume, it will be found there stated that the notes were written at the time of reading the volumes, as they were successively published. The translation was made between the years 1814 [misprinted 1815] and 1817, at which time the four first volumes, with the several appendices and notes, were ready for publication." The fifth volume, published by Laplace, twenty years after the others, was never translated by Dr. Bowditch, though he wrote many important notes upon it.† It was his intention, however, had he lived, to translate the volume. Death has forever defeated that intention. The work which he had so nearly completed, no one lives to finish as he would have finished it; but like the beautiful painting, from which was taken the accompanying engraving, and which never received the final touch of the dying artist, it is more interesting from the circumstance under which it was left incomplete.

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* Baron Zach, in his *Correspondance Astronomique*, Vol. X., p. 234, A. D. 1824, says, "Nous finérons cette note par apprendre à nos lecteurs ce que nous a révélé le professeur Everett, que M. Bowditch a traduit en anglais toute la Mécanique Céleste de M. Laplace, avec un ample commentaire, mais qu'on n'a pu encore le persuader de publier cet ouvrage, qui ne pourrait que lui faire un honneur infini, ainsi qu' à son pays, mais nous soupçonnons qu' il attend pour cela l'ouvrage de MM. Plana et Carlini, qui est sur le métier, et qui ne tardera pas à paraitre." A similar public announcement of this fact had been made in the North American Review for April, 1820, Vol. X., p. 272; and the editor says that Dr. Bowditch "has not, however, yet been prevailed upon to do honor to himself and to his country, by the publication of so great and arduous a work."

† A day or two only before his death, he received from Europe a translation, executed by a young lady whom he had never seen, but who was soon to become his daughter, embracing in seventy manuscript pages, the first part of the fifth volume; — a suitable offering of filial duty to one who never lived to thank her in person for her kindness, but who left for her at his decease, an affectionate letter written exactly a week before his death.

This valuable *résumé* of the scientific and literary labors of Dr. Bowditch is taken from his Memoir, in the fourth volume of Mécanique Céleste, written by N. I. Bowditch, Esq.; to whose kindness the readers of the Monthly are indebted for the beautiful portrait of his distinguished father which graces this number. — Ed.

Mathematical Monthly Notices.

Lessons Introductory to the Modern Higher Algebra. By Rev. George Salmon, A. M., Fellow and Tutor, Trinity College, Dublin. pp. 147, 8vo. Dublin: Hodges, Smith & Co., Grafton Street, Booksellers to the University. 1859.

It is well known to our readers that during the last few years, there has grown up, mostly by the labors of English mathematicians, what may be called a new department of analysis, and what our author terms the "Modern Higher Algebra," or the "Algebra of Linear Transformations, a department of Analysis which has been created during the last twenty years." The papers upon different points in this new analysis, which come to us through "The Quarterly Journal of Pure and Applied Mathematics," "The Memoirs of the Royal Society," and the other various European Journals and Transactions, presuppose so much knowledge, that all, but the very few, have passed them by without making any effort to read them. It is therefore with the deepest satisfaction, that we are able to announce a purely elementary work, prepared by one whose ability to do justice to the undertaking is so fully shown in his "Conic Sections" and "Higher Plane Curves," as a "guide to this branch of Algebra." We must for the present content ourselves with promising, in future numbers of the Monthly, so much of the volume before us as will define the technical terms used in the new analysis, a list of which we append.

Bezoutiant.	Canonical Form.	Canonizant.
Congredient.	Contragredient.	Combinants.
Commutants.	Concomitant.	Mixed Concomitant.
Contravariant.	Covariant.	Determinant.
Dialytic.	Discriminant.	Eliminant.
Emanant.	Evectant.	Hessian.
Hyperdeterminant.	Invariant.	Jacobian.
Minor.	Quantic.	Reciprocal of a Determinant.
Reciprocal.	Substitutions.	Singular Roots.
Symmetrical Determinant.	Umbral.	Weight.

The term *quantic* denotes a homogeneous function in general, and the words *quadric*, *cubic*, *quartic*, *quintic*, *nic*, denote quantities of the 2nd, 3rd, 4th, 5th, *n*th degrees. Quantics are also distinguished into binary, ternary, quartenary, *n*-ary, according as they contain 2, 3, 4, *n* variables. There are other subsidiary terms which it is not necessary to give in this connection.

Our author says, "To A. Cayley, Esq., and J. J. Sylvester, Esq., I beg to inscribe this attempt to render some of their discoveries better known, in acknowledgment of the obligations I am under, not only to their published writings, but also to their instructive correspondence." We trust that this volume may be the means of introducing American students into this new and fertile field of research. We commend it to their attention.

A Treatise on Problems of Maxima and Minima, solved by Algebra. By Ramchundra, late Teacher of Science, Delhi College. Calcutta: Printed by P. S. D'Rozario & Co., Tank Square, 1850, pp. 180. Reprinted by order of the Honorable Court of Directors of the East India Company for circulation in Europe and in India, in acknowledgment of the merits of the author, and in testimony of the sense entertained of the importance of independent

speculation as an instrument of national progress in India. Under the superintendence of AUGUSTUS DE MORGAN, F. R. A. S., F. C. P. S., of Trinity College, Cambridge; Professor of Mathematics in University College, London. London: WM. H. ALLEN & Co., 7 Leadenhall St., 1859.

An extended notice of this interesting work will hardly be necessary, in addition to the examples already given of our author's method, on page 41. Chapter I. contains solutions of 55 problems, involving equations of the first and second degrees; Chapter II. 27 of the third degree; Chapter III. 18 of the fourth, fifth, sixth and seventh degrees; Chapter IV. 18, involving two or more variable quantities; the whole forming a most interesting collection of problems in maxima and minima, every one of which is solved by the methods used in the examples we have quoted. Prof. DE MORGAN'S Preface is exceedingly interesting, and we regret not to be able to lay it in full before our readers. After giving some account of the correspondence which led the Court of Directors of the Honorable East India Company to concur in his views, the Editor says: "I shall at once proceed to a short account of these views; after which I shall give some account of RAMCHUNDRA, the author of the work. Of course it will be remembered that the late Court of Directors is in no way answerable for the details of my exposition, though their decided approbation was bestowed on the general sketch which I laid before them.

There are many persons, even among those who seriously turn their thoughts to the improvement of India, who look upon the native races as men to be dealt with in the same manner as Caffres or New Zealanders. Judging by the lower races of the Peninsula, and judging even these more by the grosser parts of their mythology than by the state of domestic life and hereditary institutions, they presume that the Indian question resolves itself into an inquiry how to create a mind in the country, and that mind fashioned on the English standard. They forget that at this very moment there still exists among the higher castes of the country — castes which exercise vast influence over the rest — a body of literature and science which might well be the nucleus of a new civilization, though every trace of Christian and Mahomedan civilization were blotted out of existence. They forget that there exists in India, under circumstances which prove a very high antiquity, a philosophical language which is one of the wonders of the world, and which is a near collateral of the Greek, if not its parent form. From those who wrote in this language we derive our system of arithmetic, and the algebra which is the most powerful instrument of modern analysis. In this language we find a system of logic and of metaphysics; an astronomy worthy of comparison with that of Greece in its best days; above comparison, if some books of PTOLEMY'S Syntaxis be removed. We find also a geometry, of a kind which proves that the Hindoo was below the Greek as a geometer, but not in that degree in which he was above the Greek as an arithmetician. Of the literature, poetry, drama, &c., which flourished in union with this science, I have not here to speak.

Those who consult COLEBROOKE'S translation of the *Vija Ganita*, or the account given of it in the *Penny Cyclopædia*, will see that I have not exaggerated the point most connected with this preface.

"Greece and India stand out, in ancient times, as the countries of indigenous speculation. But the intellectual fate of the two nations was very different. Among the Greeks, the power of speculation remained active during their whole existence as a nation, even down to the taking of Constantinople: it declined, indeed, but it was never extinguished. Their latest knowledge was inquisitive, as well as their earliest. They preserved their great writers unabridged and unaltered and EUCLID did not degenerate into what are called *practical rules*.

"In India, speculation died a natural death. A taste for *routine* — a thing to which inaccurate thinkers give the name of *practical* — converted their system into a collection of rules and results. Of this character are all the mathematical books which have been translated into English; per-

haps all which still exist. That they must have had an extensive body of demonstrated truths is obvious; that they lost the power and the wish to demonstrate is certain. The Hindoo became, to speak of the highest and best class, the teacher of results which he could not explain, the retailer of propositions on which he could not found thought. He had the remains of ancestors who had investigated for him, and he lived on such comprehension of his ancestors as his own small grasp of mind would allow him to obtain. He fed himself and his pupils upon the chaff of obsolete civilization, out of which Europeans had thrashed the grain for their own use.

"But the mind thus degenerated is still a mind; and the means of restoring it to activity differ greatly from those by which a barbarous race is to be gifted with its first steps of progress. No man alive can, on sufficient data, reason out the restoration of a decayed national intellect, possessed of a system of letters and science which has left nothing but dry results, inveterate habits of routine, great reverence for old teachers, and small power of comprehending the very teaching which is held in traditional respect. And this because the question is now tried for the first time. Many friends of education have proposed that Hindoos should be fully instructed in English ideas and methods, and made the media through which the mass of their countrymen might receive the results in their own languages. Some trial has been given to this plan, but the results have not been very encouraging in any of the higher branches of knowledge. My conviction is, that the Hindoo mind must work out its own problem; and that all we can do is to *set it to work;* that is, to promote independent speculation on all subjects by previous encouragement and subsequent reward. This is the true plan; all others are neither fish nor flesh.

"The history of England, as well as of other countries, having impressed me with a strong conviction that pure speculation is a powerful instrument in the progress of a nation, and my own birth and descent having always given me a lively interest in all that relates to India, I took up the work of RAMCHUNDRA with a mingled feeling of satisfaction and curiosity: a few minutes of perusal added much to both. I found in this dawn of the revival of Hindoo speculation two points of character belonging peculiarly to the Greek mind, as distinguished from the Hindoo; one of which may have been fostered by the author's European teachers, but certainly not the other.

"The first point is a leaning towards geometry. Persons who are not mathematicians imagine that all mathematicians are for all mathematics. Nothing can be more erroneous. Not merely have the two great branches, geometry and algebra, their schools of disciples, each of which looks coldly upon the other; but even geometry itself, and algebra itself, have subdivisions of which the same thing may be said. For example, Mr. DRINKWATER-BETHUNE, above mentioned, was by taste an *algebraist;* as a practised eye would at once detect from his unfinished work on equations. Business brought him to my house one morning, nearly thirty years ago, at a time when I happened to be studying some of the geometrical developments of the school of MONGE. On my pointing out to him some of the most remarkable of the conclusions, he said, with a smile, "I see that sort of thing has charms for you." Now the Hindoo was also an algebraist, as decidedly as the Greek was a geometer; the first sought refuge from geometry in algebra, the second sought refuge from arithmetic in geometry. The greatness of Hindoo invention is in algebra, the greatness of Greek invention is in geometry. But RAMCHUNDRA has a much stronger leaning towards geometry than could have been expected by a person acquainted with the *Vija Ganita;* but he has not the power in geometry which he has in algebra. I have left one or two failures — one very remarkable — unnoticed, for the reader to find out. Should this preface — as I hope it will — fall into the hands of some young Hindoos who are systematic students of mathematics, I beg of them to consider well my assertion that their weak point must be strengthened by the cultivation of pure geometry. EUCLID must be to them what BHASCARA, or some other algebraist, has been to Europe.

"The second point is yet more remarkable. Greek geometry, as all who have read EUCLID may guess, gained its strength by *striving against self-imposed difficulties*. It was not permitted to take instruments from every conception which the human mind could form; definite limitation of means was imposed as a condition of thought, and it was sternly required that every feat of progress should be achieved by those means, and no more. Just as the Greek architecture studied the production of rich and varied effect out of the simplest elements of form, so the Greek geometry aimed at the demonstration of all the relations of figure on the smallest amount of postulated basis. The great problem of squaring the circle, now with good reason held in low esteem, was the struggle of centuries to bring under the dominion of the prescribed means what might with the utmost ease have been conquered by a very small additional allowance The attempt was unsuccessful; so was that of COLUMBUS to discover India from the west. But COLUMBUS commenced the addition of America to the known world; and in like manner the squarers of the circle, and their refuters, added field on field to the extent of geometry, and aided largely in the preparation for the modern form of mathematics. Very few of these additions would have been made, at or near the time when they were made, if it had satisfied the Greek mind to meet each difficulty, as it occurred, by permission to use additional assumptions in geometry.

"The remains of the Hindoo algebra and geometry show to us no vestige of any attempt to gain force of thought by struggling against limitation of means: this, of course, because their mode of demonstration does not appear in the works which are left, or at least in those which have become known to Englishmen. But we have here a native of India who turns aside, at no suggestion but that of his own mind, and applies himself to a problem which has hitherto been assigned to the differential calculus, under the condition that none but purely algebraic process shall be used. He did not learn this course of proceeding from his European guides, whose aim it has long been to push their readers into the differential calculus with injurious speed, that they may reach the full application of mathematics to physics; and who often allow their pupils to read EUCLID with eyes shut to his limitations. RAMCHUNDRA proposed to himself a problem which a beginner in the differential calculus masters with a few strokes of the pen in a month's study, but which might have been thought hardly within the possibilities of pure algebra. His victory over the theory of the difficulty is complete. Many mathematicians of sufficient power to have done as much would have told him, when he first began, that the end proposed was perhaps unattainable by any amount of thought; next, that when attained, it would be of no use. But he found in the demands of his own spirit an impulse towards speculation of a character more fitted to the state of his own community than the imported science of his teachers. He applied to the branch of mathematics which is indigenous in India, the mode of thought under which science made its greatest advances in Greece. My own strong suspicion that it was the want of this mode of thought which allowed the decline of algebra in ancient India, coupled with my thorough conviction that, whether or no, this mode of thought yields the proper nutriment for mathematical science in its early and feeble life, produced the recommendation to the Court of Directors to which this reprint owes its own existence.

"RAMCHUNDRA'S problem — and I think it ought to go by that name, for I cannot find that it was ever current* as an exercise of ingenuity in Europe — is to find the value of a variable which will make an algebraical function a maximum or a minimum, under the following condi-

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* It would not at all surprise me if it should be found that some one inquirer has suggested the problem; but, if so, I think the search which I have made entitles me to say that the suggestion entirely failed to attract attention, and to establish the difficulty as a recognized exercise.

tions. Not only is the differential calculus to be excluded, but even that germ of it which, as given by FERMAT in his treatment of this very problem, made some think that he was entitled to claim the invention. The values of ϕx and of $\phi(x+h)$ are not to be compared; and no process is to be allowed which immediately points out the relation of ϕx to the derived function $\phi' x$. A mathematician to whom I stated the conditioned problem made it, very naturally, his first remark, that he could not see how on earth I was to find out when it would be biggest, if I would not let it grow. The mathematician will at last see that the question resolves itself into the following: — Required a constant, r, such that $\phi x - r$ shall have a pair of equal roots, without assuming the development of $\phi(x+h)$ or any of its consequences.

"RAMCHUNDRA, the author of this work, has transmitted to me some notes of his own life, from which I collect, as follows. He was born in 1821, at Paneeput, about fifty miles from Delhi. His father, SOONDUR LALL, was a Hindoo Kaeth, and a native of Delhi, and was there employed under the collector of the revenue. He died at Delhi, in 1831-2, leaving a widow (who still survives) and six sons. After some education in private schools, RAMCHUNDRA entered the English Government school at Delhi, to every pupil of which two rupees a month were given, and a scholarship of five rupees a month to all in the first and second classes. In this school he remained six years. It does not appear that any particular attention was paid to mathematics in this school; but, shortly before leaving it, a taste for that science developed itself in RAMCHUNDRA, who studied at home with such books as he could procure. After leaving school, he obtained employment as a writer for two or three years. In 1841, changes took place in the educational department of the Bengal presidency; the school was formed into a college; and RAMCHUNDRA obtained, by competition, a senior scholarship, with thirty rupees a month. In 1844, he was appointed teacher of European science in the Oriental department of the college, through the medium of the vernacular, with fifty rupees a month additional. A vernacular translation society was instituted, and RAMCHUNDRA, in aid of its object, translated or compiled works in Oordoo, and also on algebra, trigonometry, &c., up to the differential and integral calculus."

A Treatise on Differential Equations. By GEORGE BOOLE, F. R. S., Professor of Mathematics in the Queen's University, Ireland, Honorary Member of the Cambridge Philosophical Society. 485 pp. Crown, 8vo. Cambridge: MACMILLAN & Co., and 23 Henrietta Street, Covent Garden, London. 1859.

The works devoted exclusively to a systematic treatment of Differential Equations, are neither so many, nor so exhaustive, that teachers and students will not welcome another, provided it will be well adapted to the wants of elementary instruction. To give the author's idea of these wants, and how they are to be supplied, we give the following extract from the Preface.

"It was my object first of all, to meet the wants of those who had no previous acquaintance with the subject, but I also desired not quite to disappoint others who might seek for more advanced information. These distinct, but not inconsistent aims determined the plan of composition. The earlier sections of each chapter contain that kind of matter which has usually been thought suitable for the beginner, while the latter ones are devoted either to an account of recent discovery, or to the discussion of such deeper questions of principle as are likely to present themselves to the reflective student in connection with the methods and processes of his previous course. The principles which I have kept in view in carrying out the above design are the following:

"1st. In the exposition of methods I have adhered as closely as possible to the historical order of their development. I presume that few who have paid any attention to the history of the Mathematical Analysis, will doubt that it has been developed in a certain order, or that that order has been, to a great extent, necessary — being determined, either by steps of logical de-

duction, or by the successive introduction of new ideas and conceptions, when the time for their evolution had arrived. And these are causes which operate in perfect harmony. Each new scientific conception gives occasion to new applications of deductive reasoning; but those applications may be only possible through the methods and the processes which belong to an earlier stage.

"Thus, to take an illustration from the subject of the following work: The solution of ordinary simultaneous differential equations properly precedes that of linear partial differential equations of the first order; and this, again, properly precedes that of partial differential equations of the first order which are not linear. And in this natural order were these subjects developed. Again, there exists large and very important classes of differential equations, the solution of which depends on some process of successive reduction. Now such seems to have been effected, at first, by a repeated change of variables; afterwards, and with greater generality, by a combination of such transformations with others involving differentiation; last of all, and with greatest generality, by symbolical methods. I think it necessary to direct attention to instances like these, because the indications which they afford, appear to me to have been, in some work, of great ability, overlooked; and because I wish to explain my motives for departing from the precedent thus set.

"Now there is this reason for grounding the order of exposition upon the historical sequence of discovery, that by so doing we are most likely to present each new form of truth to the minds precisely at that stage at which the mind is most fitted to receive it, or even like that of the discoverer, to go forth to meet it."

The plan of composition indicated in the above extract is completely carried out in nearly every chapter, of which there are eighteen. The elementary articles of each chapter, which the beginner should first read, are indicated by the author. A valuable feature of the work is the collection of interesting examples appended to each chapter; in all, over 300. At the end of the volume are found the answers of nearly one half the problems, which will be of great convenience to the student while acquiring confidence in his knowledge of the theory. Besides a general survey of the whole volume, we have read several of the earlier chapters with care, to be able to judge of its adaptation to the wants of students in this country; and we can commend it as being the very best work we have ever seen for a text-book upon the subject of Differential Equations; nor need the mathematician fear that he will fail to find it a quite full compendium of our present knowledge of this important department of analysis.

Editorial Items.

THE following gentlemen have sent us solutions of the Prize Problems in the August number of the MONTHLY:—

DAVID TROWBRIDGE, Perry City, Schuyler Co., N. Y., answered all the questions. His solutions of the Prize Problems in the July number did not reach us in time; as was also the case with the solutions sent by JAMES M. INGALS, of Delton, Sauk Co., Wis.

ASHER B. EVANS, Madison University, Hamilton, N. Y., answered all the questions.

GUSTAVUS FRANKENSTEIN, Springfield, Ohio, answered all the questions.

James F. Roberson, Senior Class, Indiana University, Bloomington, answered question III.

We have also received a set of excellent solutions from Asaph Hall, Esq., Assistant at Harvard College Observatory, who does not wish to compete for the prizes.

Prof. Werden Reynolds, late Principal of the Worcester Academy, has accepted the Presidency of the Worcester Female College. . . . . Prof. B. S. Hedrick has been appointed to the Professorship of Mathematics in Cooper Institute, N. Y. . . . . Rev. Thomas Hill will deliver a course of lectures before the Lowell Institute, in Boston, in a few weeks, and return to Yellow Springs, Ohio, about January 1, 1860. . . . . Prof. Chauvenet, of the U. S. Naval Academy at Annapolis, Md., has accepted a Professorship in Washington University, at St. Louis, Mo. . . . . William Watson, Esq., a graduate of and Tutor in the Lawrence Scientific School, already well and favorably known to our readers, will sail for Paris on the 12th of November, where he proposes to spend a year or more in the study of the Mathematics. Mr. Watson has kindly consented to be an occasional correspondent of the Mathematical Monthly during his absence, and send us such information concerning the schools and methods of instruction, especially in the Mathematics, as it may be in his power to give. Instead of the hasty observations of the mere tourist, we shall be informed of every day school life abroad, by one whose education and experience as a teacher of the mathematics will enable him to judge of those things which will most benefit his fellow teachers at home.— Our best wishes go with him. . . . . It gives us pleasure to add the following names to our list of coöperators and contributors: E. A. Strong, Esq., Grand Rapids, Mich.; Henry Ward Poole, Esq., Boston; Lucius Brown, Esq., Fall River, Mass.; Saxe Gotha Laws, Esq., Dover, Delaware; Chauncey Smith, Esq., Boston.

The portrait of Dr. Bowditch, which we have the gratification of presenting to our readers, needs no commendation at our hands, as a work of art; and we will also add that it is recognized by his family as a faithful likeness. It is eminently proper that the first portrait given in the Monthly should be of the "Father of American Geometry." We commend the list of his writings, with the interesting and valuable series of notes by his son, N. I. Bowditch, Esq., to the attention of all. We trust it may not be long before we shall have the pleasure of presenting the portraits of other eminent mathematicians, both of our own and foreign countries. . . . . Books Received. Tables of Victoria, Computed with regard to the Perturbations of Jupiter and Saturn. By F. Brünnow, Ph. Dr., Professor of Astronomy in the University of Michigan, and Director of the Observatory at Ann Arbor. Printed by order of the Board of Regents. New York: B. Westermann & Co. London: Trübner & Co. 4to. pp. 75. 1859. . . . . Linear Perspective Explained. By William N. Bartholomew, Author of Bartholomew's Sketch Book, and Series of Drawing Books, in six numbers. Boston: Shepard, Clark & Brown. 1859. . . . . Résumé de Leçons de Géométrie Analytique et de Calcul Infinitésimal. By J. B. Belanger, Ingénieur en chef des Ponts et Chaussées, Professeur de Mécanique à l'Ecole Impériale Polytechnique et à l'Ecole Centrale des Arts et Manufactures. Seconde Edition. Paris: Mallet-Bachelier. 1859. . . . . Journal de Mathématiques Pures et Appliquées; Publié by Joseph Liouville. Juillet. 1859. . . . . Nouvelles Annales de Mathématique Rédigé Par MM. Terquem et Gerono. Septembre, 1859.

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